Math 250 – Sect.2.4: The Chain Rule

I. Review - composition of functions.

A composite function is one which is composed of (or built up from) simpler functions.

-example- $y=(3x-4)^3$ can be thought of as a composite function, $f[g(x)]$, where

$f(x)=$ $g(x)=$

-examples- For each function, identify the OUTER (*f*) and INNER(*g*) functions for the composition.

1. $y=\sqrt{4-x^2}$ 2. $y=\sin^3 x$ 3. $y=\dfrac{4}{\sqrt[3]{x+5}}$

II. A new differentiation rule.

CHAIN RULE: $\dfrac{d}{dx}f[g(x)]=f'[g(x)]\cdot g'(x)$

OR $\dfrac{d}{dx}f[u]=f'[u]\cdot du$

This rule is applied to derivatives of COMPOSITE FUNCTIONS.

-example- Consider the function: $y=\left(x^2+3x-4\right)^7$

a. Identify the inner function - $g(x)=$ ____________________ (this is also called u)

b. Identify the outer function - $f(x)=$ __________________

OR $f(u)=$ ________________

c. Apply the chain rule to find *dy/dx*.

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All of our previously learned rules can now be generalized:

1. The Power Rule: $\frac{d}{dx}[x^n] =$ ______________ or $\frac{d}{dx}[u^n] =$ _____________________

2. The Trigonometric Functions:

a. $\frac{d}{dx}[\sin x] =$ ___________ or $\frac{d}{dx}[\sin u] =$ _______________________

b. $\frac{d}{dx}[\cos x] =$ ___________ or $\frac{d}{dx}[\cos u] =$ _______________________

c. $\frac{d}{dx}[\tan x] =$ ___________ or $\frac{d}{dx}[\tan u] =$ _______________________

d. $\frac{d}{dx}[\sec x] =$ ___________ or $\frac{d}{dx}[\sec u] =$ _______________________

e. $\frac{d}{dx}[\csc x] =$ ___________ or $\frac{d}{dx}[\csc u] =$ _______________________

f. $\frac{d}{dx}[\cot x] =$ ___________ or $\frac{d}{dx}[\cot u] =$ _______________________

-examples- Find the derivative for each of the following.

1. $y = (3x-4)^5$

2. $y = \sqrt{6-x^2}$

3. $P(t) = \dfrac{5}{2t+1}$

4. $N(r) = \cos(7r)$

5. $f(x)=\sin^3 x$

6. $f(x)=\sin^3(8x)$

*Sometimes, we have to combine this chain rule with product or quotient rules. . .

7. $y=(7x-4)^3\sin(2x)$

8. $y=\dfrac{(5x-4)^3}{(4x+7)^2}$

9. $f(x) = x^2\sqrt{1-x^2}$

And of course . . . let's not forget some applications.

-example- Find the equation of the line tangent to the curve $f(x) = \sec(2x)$ when $x = \dfrac{\pi}{6}$.

-example- A particle moves along a horizontal line with position function $s(t) = \dfrac{4}{\sqrt{\sin t + 2}}$, where s represents the position of the particle in relation to the origin (measured in feet), and t is measured in seconds. Find the velocity function, and the velocity at time $t = 1$ and $t = 2$.